What is nonclassical about uncertainty relations?

Gdańsk - 03/08/2022

Lorenzo Catani

Joint work with Matt Leifer, Giovanni Scala, David Schmid and Rob Spekkens

arXiv:2207.11779

Technische Universität Berlin

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Motivation

- The basic phenomenology of features usually considered as truly nonclassical is exhibited in theories admitting of a noncontextual ontological model.

Motivation

Phenomena arising in Spekkens' toy theory	Phenomena not arising in Spekkens' toy theory
Noncommutativity	Bell inequality violations
Coherent superposition	Noncontextuality inequality violations
Collapse	Computational speed-up (if it exists)
Complementarity	Certain aspects of items on the left
No-cloning	
No-broadcasting	
Teleportation	
Remote steering	
Key distribution	
Dense coding	
Entanglement	
Monogamy of entanglement	
Choi-Jamiolkowski isomorphism	
Naimark extension	
Stinespring dilation	
Ambiguity of mixtures	
Locally immeasurable product bases	
Unextendible product bases	
Pre and post-selection effects	
Interference	
Elitzur-Vaidman bomb tester	
Wheeler's delayed-choice experiment	
Quantum eraser	
And many others...	

R. W. Spekkens, in Quantum Theory: Informational Foundations and Foils, pp 83-135, Springer Dordrecht (2016).

*L. Catani, M. Leifer, D. Schmid and R.W. Spekkens, arXiv:2111.13727 (2021).

Motivation

- The basic phenomenology of features usually considered as truly nonclassical is exhibited in theories admitting of a noncontextual ontological model.

- Which aspects of those phenomena witness contextuality?

[8] D. Schmid and R. W. Spekkens, Phys. Rev. X **8**, 011015 (2018).
[9] K. Flatt, H. Lee, C. R. i. Carceller, J. B. Brask, and J. Bae, arXiv preprint arXiv:2112.09626 (2021).
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[11] M. Lostaglio and G. Senno, Quantum **4**, 258 (2020).
[12] M. F. Pusey and M. S. Leifer, arXiv preprint arXiv:1506.07850 (2015).
[13] M. F. Pusey, Phys. Rev. Lett. **113**, 200401 (2014).
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Motivation

- There exist theories that manifest uncertainty relations but also admit of a noncontextual ontological model.

- Which aspects of uncertainty relations witness contextuality?

Contents

"Product" uncertainty relations in quantum theory

W. Heisenberg (1901-1976)

H.P. Robertson (1903-1961)

expectation value of commutator

$$\Delta A^2 \Delta B^2 \geq \left| \frac{\langle [A, B] \rangle}{2} \right|^2$$

variances

Can be trivial

"Sum" uncertainty relations in quantum theory

In the case of Pauli X and Z measurements,

$$\Delta X^2 + \Delta Z^2 \geq 1.$$

Given that $\Delta X^2 = \langle X^2 \rangle - \langle X \rangle^2 = 1 - \langle X \rangle^2$ and $\Delta Z^2 = \langle Z^2 \rangle - \langle Z \rangle^2 = 1 - \langle Z \rangle^2$,

$$\langle X \rangle^2 + \langle Z \rangle^2 \leq 1.$$

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- Motivation
- Uncertainty relations
- Operational theories
- Ontological models and Noncontextuality
- Main result
- Conclusion

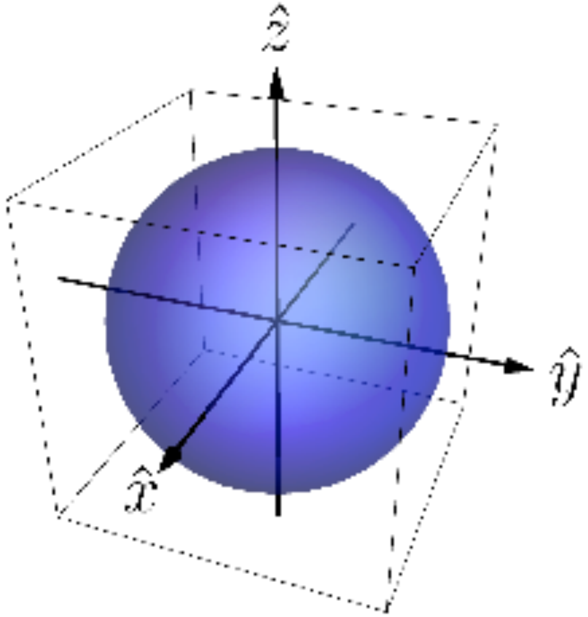
Operational theories

Operational theory in a prepare and measure scenario:

Preparation P	$\longleftrightarrow$	$\vec{s}_P$
Measurement and outcome $M,\ y$	$\longleftrightarrow$	$\vec{e}_{y\|M}$
Predicted statistics $\mathbb{P}(y\|M, P)$	$\longleftrightarrow$	$\vec{s}_P \cdot \vec{e}_{y\|M}$

Operational theories – examples

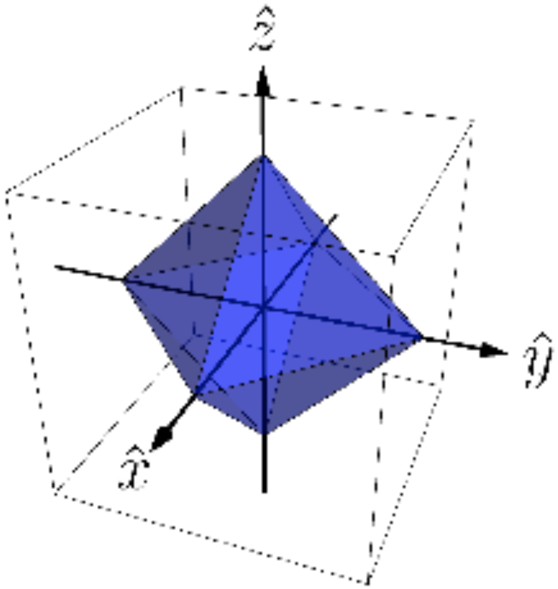
Qubit theory



$$\langle X \rangle^2 + \langle Z \rangle^2 \leq 1$$

Operational theories – examples

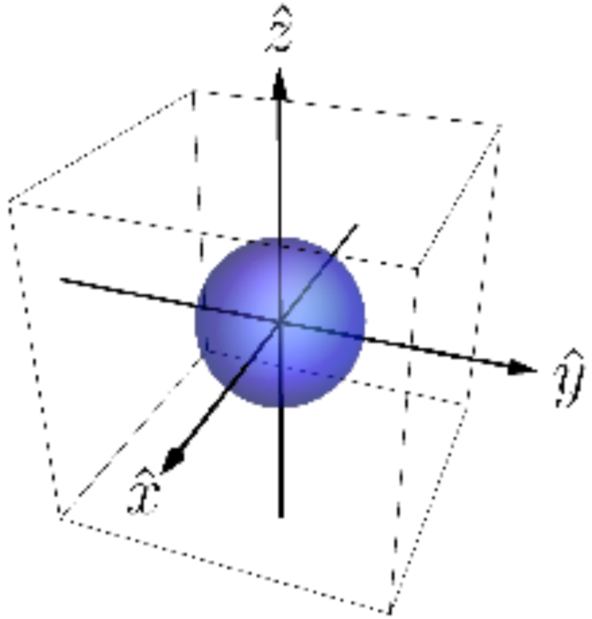
Stabilizer theory



$$|\langle X\rangle| + |\langle Z\rangle| \leq 1$$

Operational theories – examples

$\eta-$ depolarized qubit theory

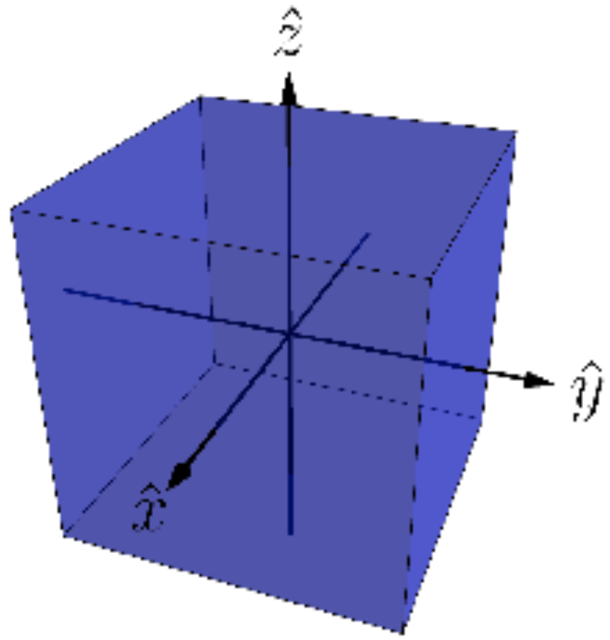


$$\mathcal{D}_\eta(\rho) \equiv (1-\eta)\rho + \eta\frac{\mathbb{I}}{2}$$

$$\langle X\rangle^2 + \langle Z\rangle^2 \leq (1-\eta)^2$$

Operational theories – examples

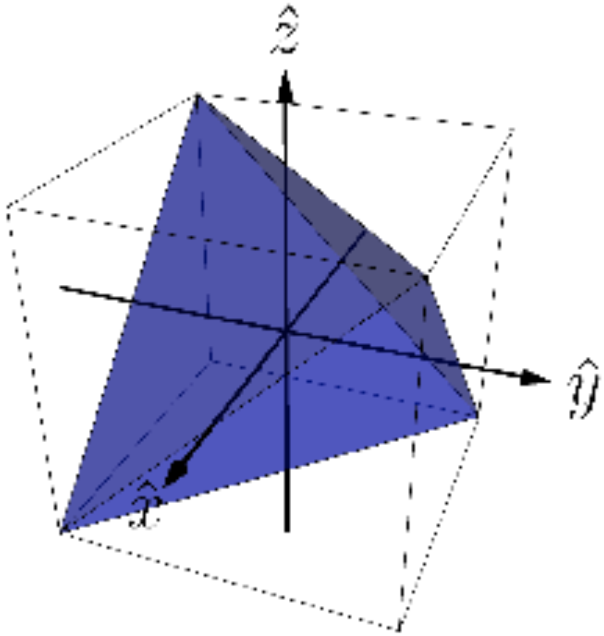
Gbit theory



$$|\langle X\rangle| \leq 1, |\langle Z\rangle| \leq 1$$

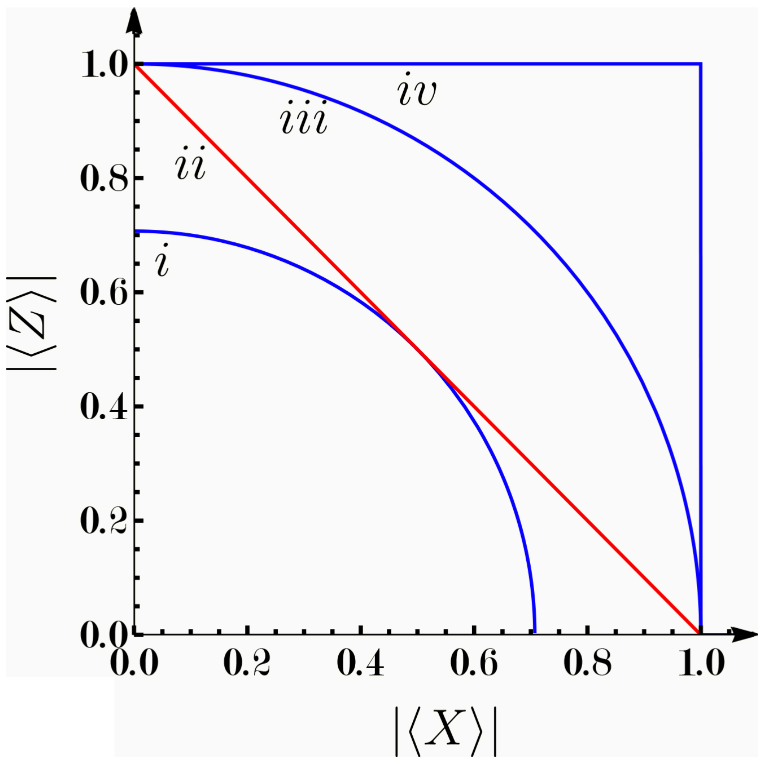
Operational theories – examples

Simplicial theory



$$|\langle X\rangle| \leq 1, |\langle Z\rangle| \leq 1$$

Examples: comparison of uncertainty relations



Legend

$i = (1 - \frac{1}{\sqrt{2}})$−depolarized qubit theory
ii = stabilizer theory
iii = qubit theory
iv = gbit theory

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Ontological model of an operational theory

Each system ⟶ ontic state space Λ describing possible ontic states $\lambda \in \Lambda$.

Preparation P ⟶ $\mu(\lambda|P)$ ⟷ $\vec{\mu}_P$

Measurement and outcome $M,\ y$ ⟶ $\xi(y|M,\lambda)$ ⟷ $\vec{\xi}_{y|M}$

Predicted statistics $\mathbb{P}(y|M,P) = \sum_{\lambda\in\Lambda} \xi(y|M,\lambda)\mu(\lambda|P)$ ⟷ $\vec{\xi}_{y|M} \cdot \vec{\mu}_P$

Preparation noncontextuality

- Two preparations P, P' are *operationally equivalent*, $P \simeq P'$, if
  $\mathbb{P}(y|M,P) = \mathbb{P}(y|M,P') \ \ \forall M.$

- In a preparation noncontextual ontological model,

$$P \simeq P' \implies \mu(\lambda|P) = \mu(\lambda|P').$$

- In particular,

$$\sum_i w_i \vec{s}_i = \sum_j w_j' \vec{s}_j\,' \implies \sum_i w_i \vec{\mu}_i = \sum_j w_j' \vec{\mu}_j\,'.$$

R.W. Spekkens, Phys. Rev. A **71**, 052108 (2005)

Why is it a good notion of classicality?

- Instance of Leibnizian methodological principle / no-fine tuning.
- Connected to locality and Kochen-Specker noncontextuality.
- Connected to positivity of quasiprobability representations.
- Connected to simplex embeddability in GPTs.
- Emerges in the presence of sufficient noise.
- It is empirically testable.
- Its violation is connected to quantum computational advantages.

*watch David Schmid's lectures at Solstice of Foundations or see references in our article.

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Main result

How to link uncertainty relations and contextuality?

Problem:

Uncertainty relations ⟶ single state.

Contextuality ⟶ requires operational equivalences (at least 4 states).

Main result

How to link uncertainty relations and contextuality?

Solution:

Consider uncertainty relations for a state that satisfies the condition of A_1^2-orbit-realizability.

The A_1^2-orbit-realizability condition

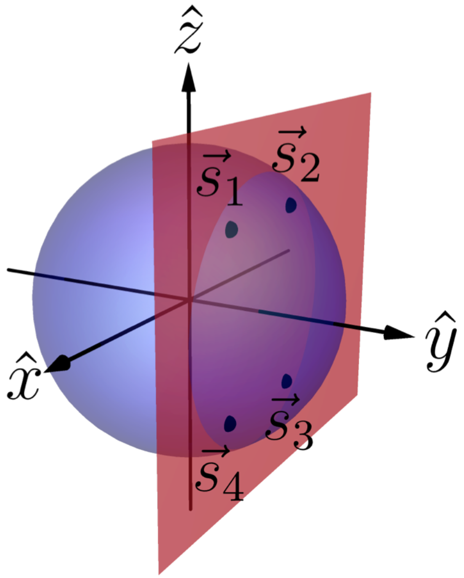
1. The state has equal predictability counterparts.

2. The state manifest operational equivalences with such counterparts.

Example in the qubit theory:

1. $\langle X\rangle_{\vec{s}_1} = -\langle X\rangle_{\vec{s}_2} = -\langle X\rangle_{\vec{s}_3} = \langle X\rangle_{\vec{s}_4},$
   $\langle Z\rangle_{\vec{s}_1} = \langle Z\rangle_{\vec{s}_2} = -\langle Z\rangle_{\vec{s}_3} = -\langle Z\rangle_{\vec{s}_4}.$

2. $\frac{1}{2}\vec{s}_1 + \frac{1}{2}\vec{s}_3 = \frac{1}{2}\vec{s}_2 + \frac{1}{2}\vec{s}_4.$



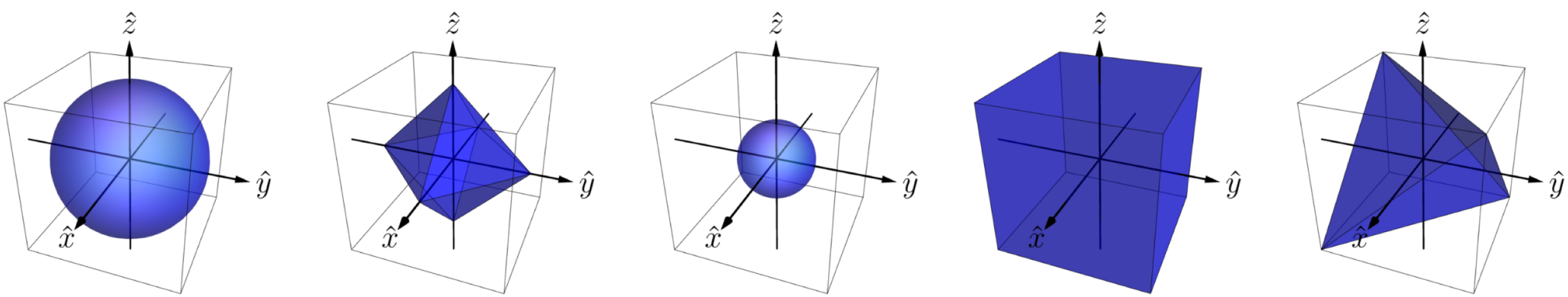
Main result

In any operational theory, if one can find a pair of measurements, X,Z , and a state that satisfies the condition of $A_1^2\text{-orbit-realizability}$, then noncontextuality implies that $|\langle X\rangle| + |\langle Z\rangle| \leq 1$.

Theories with $A_1^2-\text{symmetry}$

If *all* states in an operational theory satisfy the condition of $A_1^2\text{-orbit-realizability}$ we say that the theory has $A_1^2-\text{symmetry}$.

Examples:



(a) qubit theory ✓

(b) stabilizer theory ✓

(c) η-depolarized qubit theory ✓

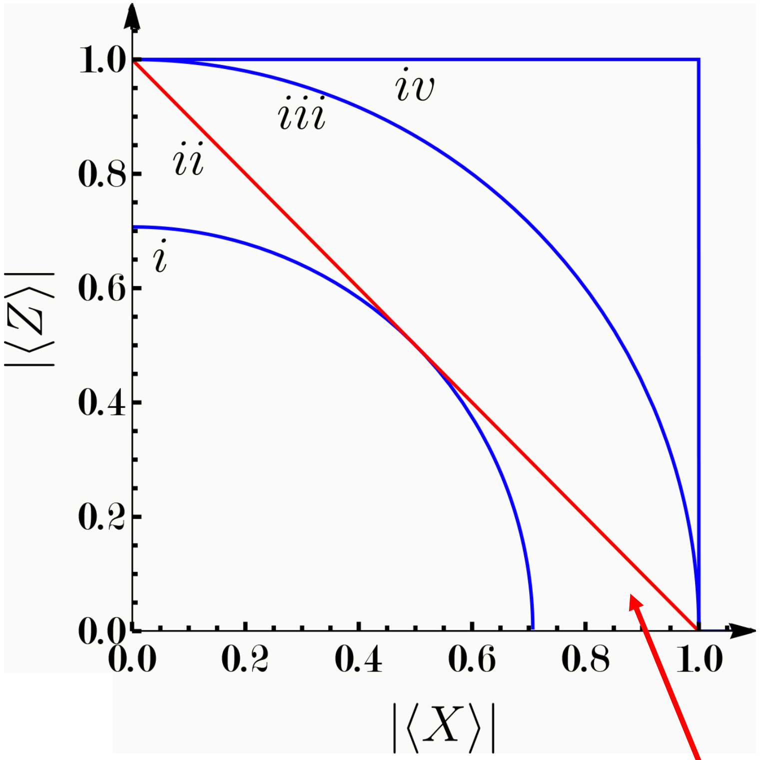
(d) gbit theory ✓

(e) simplicial theory ✗

Noncontextuality and uncertainty relations

For theories that have $A_1^2-\mathrm{symmetry}$ our bound is a constraint on the form of the *ZX-uncertainty relation* within such theories.

Examples: comparison of uncertainty relations



$|\langle X\rangle| + |\langle Z\rangle| \leq 1$

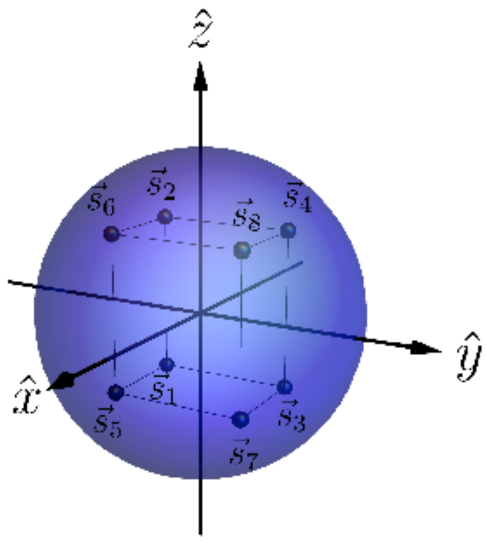
Legend

$i = (1 - \frac{1}{\sqrt{2}})$−depolarized qubit theory
ii = stabilizer theory
iii = qubit theory
iv = gbit theory

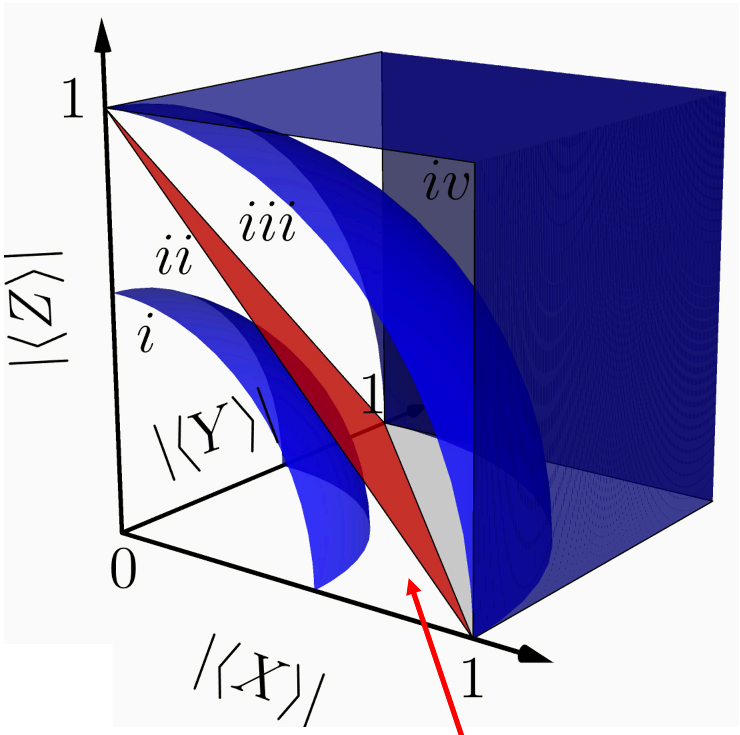
Generalization to three measurements

In any operational theory, if one can find a triple of measurements, X, Y, Z, and a state that satisfies the condition of $A_1^3\text{-orbit-realizability}$, then noncontextuality implies that $|\langle X\rangle| + |\langle Y\rangle| + |\langle Z\rangle| \leq 1.$

Example of $A_1^3\text{-orbit-realizability}$ in qubit theory:



Generalization to three measurements



$$|\langle X\rangle| + |\langle Y\rangle| + |\langle Z\rangle| \leq 1$$

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Conclusion

- Under the condition of $A_1^2\text{-orbit-realizability}$, noncontextuality bounds the functional form of the ZX predictability tradeoff below a linear curve.

- The functional form of an uncertainty relation can witness contextuality.

- If one takes noncontextuality as the notion of classicality, it is not the lack of perfect joint ZX predictability that witnesses nonclassicality, but the *greater* joint predictability for states satisfying $A_1^2\text{-orbit-realizability}$.

- Follow-up work: what is nonclassical about interference phenomena?

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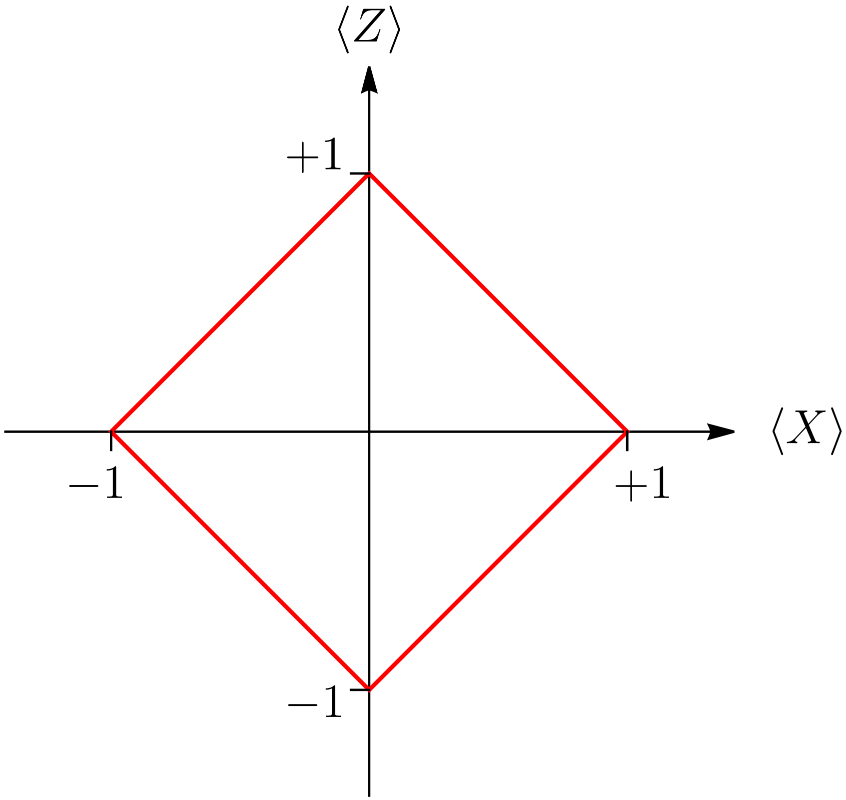
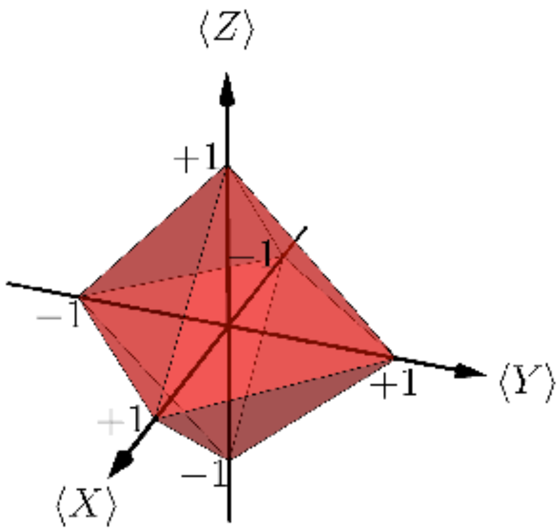
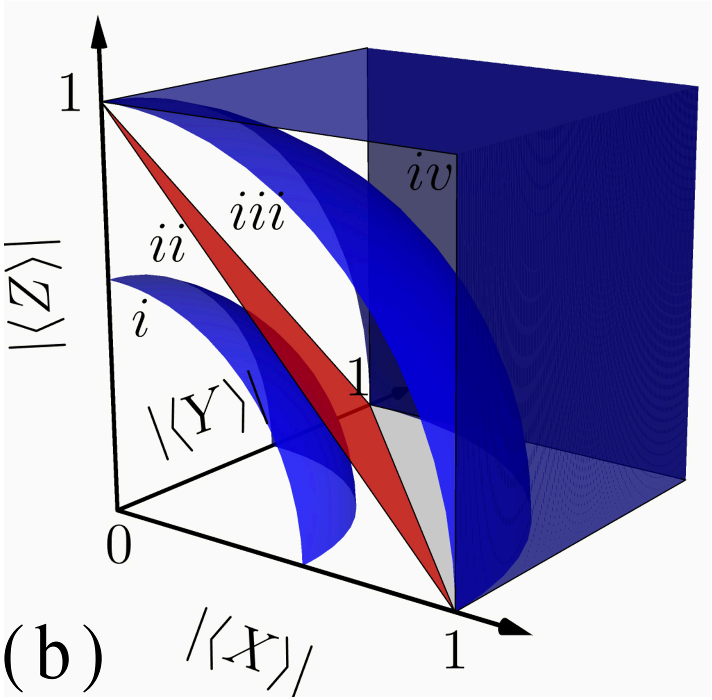
Extra slides

The noncontextual bound



The case of three measurements





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L. Catani, M. S. Leifer, D. Schmid, R. W. Spekkens, arXiv: 2111.13727 (2021).

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R. W. Spekkens *Phys Rev A* **75** (3): 032110 (2007).

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L. Catani, D. E. Browne, New J. Phys. **19**, 073035 (2017).

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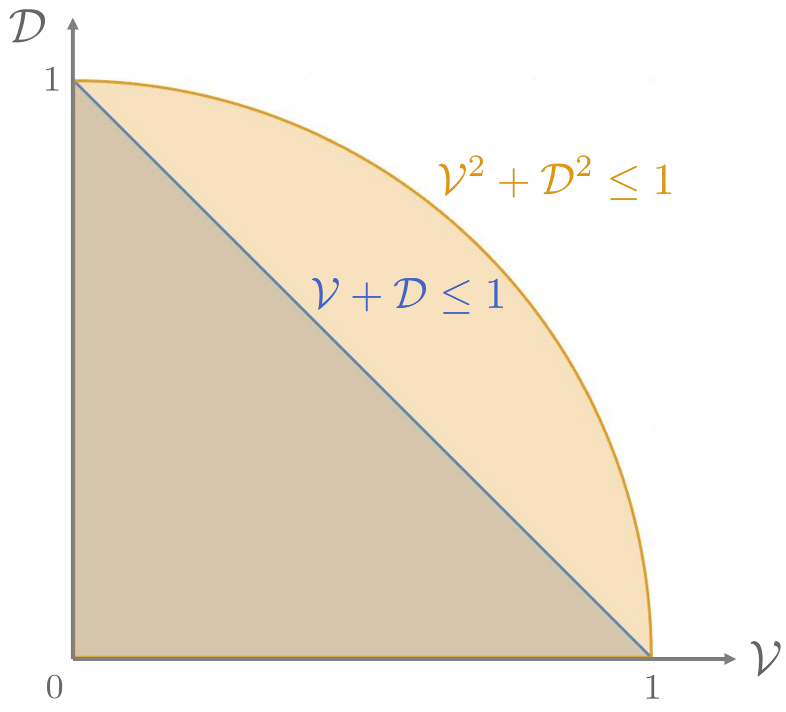
D. Schmid, J. Selby, R. W. Spekkens, arXiv:2009.03297 (2020).

What is non-classical about quantum interference?

In a follow-up work we show that the precise trade-off between visibility of fringes $\mathcal{V}$ and which-way distinguishability $\mathcal{D}$ in any preparation noncontextual model is linear, $\mathcal{V} + \mathcal{D} \leq 1$, while in quantum theory it is quadratic (Englert inequality), $\mathcal{V}^2 + \mathcal{D}^2 \leq 1$.



What is non-classical about quantum interference?

It is possible to provide a classical account of the TRAP phenomenology of quantum interference. However, reproducing the precise trade-off between visibility and distinguishability in quantum theory requires contextuality.