WHAT IS NONCLASSICAL ABOUT UNCERTAINTY RELATIONS?

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Motivation

- Uncertainty relations consist of bounds on the joint predictability of outcomes of distinct measurements on a single state.
- The existence of nontrivial uncertainty relations in quantum theory is generally considered to state a departure from the classical worldview.
- However, there exist theories that manifest uncertainty relations but also admit of a noncontextual ontological model.
- Which aspects of uncertainty relations, if any, witness contextuality?

Uncertainty relations

Definition

We consider uncertainty relations describing the tradeoff between the predictabilities of a pair of binary-outcome measurements (e.g., X and Z Paulis). Tradeoffs are in terms of sums of expectation values.



Results

In any operational theory, if one can find a pair of measurements, X,Z, and a state that satisfies the condition of A_1^2 -orbit-realizability, then noncontextuality implies that $|\langle X \rangle| + |\langle Z \rangle| \le 1$.

 $|\langle X \rangle| + |\langle Z \rangle| \le 1$

 $i = (1 - \frac{1}{\sqrt{2}})$ -depolarized qubit theory ii = stabilizer theoryiii =qubit theory iv = gbit theory $\langle X \rangle^2 + \langle Z \rangle^2 \le 1$

Condition of A_1^2 -orbit-realizability

- 1. The state has equal predictability counterparts.
- 2. The state manifest operational equivalences with such counterparts.

For theories where all states satisfy A_1^2 -orbit-realizability our bound is a constraint on the form of the ZX-uncertainty relation within such theories.

Noncontextual ontological models

Ontological model of an operational theory

- Each system \longrightarrow ontic state space Λ with ontic states $\lambda \in \Lambda$.
- Preparation $P \iff \mu(\lambda|P)$

arXiv:2207.11779

- Measurement and outcome $M, y \iff \xi(y|M, \lambda)$
- Predicted statistics $\mathbb{P}(y|M, P) = \sum \xi(y|M, \lambda)\mu(\lambda|P)$

Noncontextuality

- Two preparations P, P' are operationally equivalent, $P \simeq P'$, if $\mathbb{P}(y|M, P) = \mathbb{P}(y|M, P') \quad \forall M.$
- In a preparation noncontextual ontological model, $P \simeq P' \implies \mu(\lambda|P) = \mu(\lambda|P').$

Conclusion and future directions

- Given A_1^2 -orbit-realizability, noncontextuality bounds the functional form of the ZX predictability tradeoff below a linear curve.
- The functional form of an uncertainty relation can witness contextuality.
- What is nonclassical about interference phenomena?

References

[1] L. Catani, M. Leifer, G. Scala, D. Schmid and R.W. Spekkens arXiv:2207.11779 (2022). [2]] L. Catani, M. Leifer, D. Schmid and R. Spekkens , 2111.137272 (2021). [3] R.W. Spekkens, Phys. Rev. A 75(3): 032110 (2007). [4] R.W. Spekkens, Phys. Rev. A 71: 052108 (2005).



0.4 0.6

 $|\langle X \rangle|$

0.2

0.8 1.0

 $\overline{\langle Z \rangle}^{0.6}$

(a)

Ouantum resources: from mathematical foundations to operational characterization, December 2022 This research was made possible by funding from QuantERA/2/2020, an ERA-Net cofund in Quantum Technologies, under the project eDICT.





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