A family of multipartite separability criteria based on correlation tensor

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Abstract

A family of separability criteria based on correlation matrix (tensor) is provided. Interestingly, it unifies several criteria known e.g.realignment criterion, de Vicente criterion and derived recently separability criterion based on SIC POVMs. These new criteria are linear in the density operator and hence one may find new classes of entanglement witnesses and positive maps. Interestingly, there is a natural generalization to multipartite scenario using multipartite correlation matrix.

Definition: Trace norm of a real matrix A is defined as $\|A\|_1 = \mathrm{Tr}\sqrt{AA^T}$. for positive A: $\|A\|_1 = \mathrm{TrA}$, in general: sum of singular values,

Separability criteria by Correlator tensor

Let $\{G_i^{(k)}\}_{i=1}^{d_k^2}$ be an orthonormal hermitian basis of $\mathcal{B}(\mathbb{C}^{d_k})$
Definition: We define the *correlation tensor* of ρ as:

$$C(\rho)_{i_1,\dots,i_n} = \mathrm{Tr}\left(\rho G_{i_1}^{(1)} \otimes \cdots \otimes G_{i_n}^{(n)}\right) \tag{1}$$

if $n=2$ it is $d_1^2 \times d_2^2$ matrix of coordinates of ρ.

Known criteria

if ρ (bipartite) is separable, then

- **Realignment criterion:** $||C(\rho)||_1 \leq 1$;
- **Enhanced realignment criterion:** $||C(\rho - \rho_A \otimes \rho_B)||_1 \leq \sqrt{1-\mathrm{Tr}\rho_A^2}\sqrt{1-\mathrm{Tr}\rho_B^2}$;
- **De Vincente criterion:** $||\widetilde{C}(\rho)||_1 \leq \sqrt{1-\frac{1}{d_A}}\sqrt{1-\frac{1}{d_B}}$, where $\widetilde{C}$ is obtained removing the the first column and the first row from C.
- **ESIC criterion:** $||\hat{C}(\rho)||_1 \leq \sqrt{\frac{d_A+1}{2d_A}}\sqrt{1-\frac{d_B+1}{2d_B}}$, where $\hat{C}_{ij} = \mathrm{Tr}\left(\rho P_i \otimes Q_j\right)$, $\{P_i\}_{i=1}^{d_A^2}$ and $\{Q_i\}_{i=1}^{d_B^2}$ are SIC-POVMs.

XY-criteria

Main idea

Our criterion, redefine

$$C_{x,y} = \mathrm{diag}\{x,1,\dots,1\}C\mathrm{diag}\{y,1,\dots,1\}$$

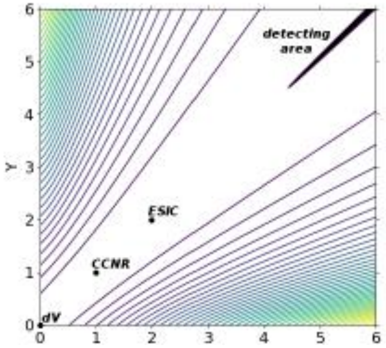
then for ρ (bipartite) separable

$$||C_{x,y}(\rho)||_1 \leq \sqrt{\frac{d_A-1+x^2}{d_A}}\sqrt{1-\frac{d_B-1+y^2}{d_B}} \tag{2}$$

As special cases we have:

- $x=y=0$ de Vincente
- $x=y=1$ realignment
- $x=y=2$ ESIC (any SIC-POVMs is required!)

For appropriately chosen $\rho \in \mathcal{B}(\mathbb{C}^3 \otimes \mathbb{C}^3)$:



Generalization of the linear criteria

What is multidimensional isometry?

Generalising to multipartite case, one has problem defining trace norm. There is no SVD and singular values for multidimensional matrices.

$$\|A\|_1 = \max_{O\in\mathcal{O}(d_1,\dots,d_n)} \langle A\rangle O_{HS} \tag{3}$$

- But what is multidimensional isometry?

Trace norm generalization

$$\|A\|_1 = \max_{M\in\mathcal{B}(\mathbb{C}^{d_1}\otimes\cdots\otimes\mathbb{C}^{d_n})} \frac{\langle A\rangle M_{HS}}{\|M\|_{sup}} \tag{4}$$

$$\|M\|_{sup} = \sup_{\substack{x_1,\dots,x_n:\\ \|x_1\|=\cdots=\|x_n\|=1}} \langle x_1\otimes\cdots\otimes x_n|M\rangle \tag{5}$$

Multipartite entanglement detection

We have proven the following: for ρ - separable:

$$\forall x_1,\dots,x_n \;\; \|C_{x_1,\dots,x_n}\|_1 \leq \prod_i \sqrt{\frac{d_i-1+x_i^2}{d_i}} \tag{6}$$

Equivalence of criteria

Enhanced realignment criterion vs. linear entanglement witnesses

$$\|C(\rho-\rho_A\otimes\rho_B)\|_1 \leq \sqrt{1-\mathrm{Tr}\rho_A^2}\sqrt{1-\mathrm{Tr}\rho_B^2}$$
$$\Rightarrow \|C_{xy}(\rho)\|_1 \leq \sqrt{\frac{d_A-1+x^2}{d_A}}\sqrt{\frac{d_B-1+y^2}{d_B}} \tag{7}$$

for all x,y. Hence no correlation tensor based criterion can detect more that the enhanced realignment criterion. Now we consider the limit witnesses:

$$W^\infty = \frac{(d_B-1)\cot\theta+(d_A-1)\tan\theta+\eta^2\sin\theta\cos\theta}{2}\frac{I_{d_A}}{\sqrt{d_A}}\otimes\frac{I_{d_B}}{\sqrt{d_B}}$$
$$+\eta\cos\theta\frac{I_A}{\sqrt{d_A}}\otimes\sum_{\beta>0}v^\beta G_\beta^B+\eta\sin\theta\sum_{\alpha>0}(\tilde{O}v)^\alpha G_\alpha^A\otimes\frac{I_B}{\sqrt{d_B}}$$
$$+\sum_{\alpha,\beta>0}\tilde{O}^{\alpha\beta}G_\alpha^A\otimes G_\beta^B \tag{8}$$

The minimum of their expected values for a given state ρ is

$$\sqrt{1-\|\rho_B\|^2}\sqrt{1-\|\rho_A\|^2}-\|\rho-\rho_A\otimes\rho_B\|_1 \tag{9}$$

Hence if the enhanced realignment criterion detects entanglement in ρ, then it is detected by a witness of a form W^∞ as well, hence it is also detected by $W_{O,x,y}$ for large enough x,y.

References

[1] G. Sarbicki, G. Scala, and D. Chruściński, *Family of multipartite separability criteria based on a correlation tensor*, Phys. Rev. A 101, 012341, (2020)

[2] G. Sarbicki, G. Scala, and D. Chruściński, *Enhanced realignment criterion vs. linear entanglement witnesses*, Journal of Physics A: Mathematical and Theoretical,10.1088/1751-8121, (2020)